Enrollment No: ____________________ Exam Seat No: ____________________

# C.U.SHAH UNIVERSITY

## Winter Examination-2021

**Subject Name: Real Analysis-I**

**Subject Code: 4SC05REA1** **Branch: B.Sc. (Mathematics)**

**Semester: 5** **Date: 13/12/2021** **Time: 11:00 To 02:00** **Marks: 70**

**Instructions:**

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1 Attempt the following questions: (14)**

**a)** Define: Field structure (02)

**b)** Find the infimum and supremum of $\left\{\frac{1}{n} : n \in N\right\}$. (01)

**c)** Define: Bounded sequence (01)

**d)** State sandwich theorem for sequence. (02)

**e)** True/False: $\sum \frac{1}{n^3}$ is divergent. (01)

**f)** Define: Cauchy Sequence, Monotonic increasing sequence (02)

**g)** True/False: Every bounded sequence is convergent. (01)

**h)** State Cauchy's root test for series. (02)

**i)** Check the series $\sum_{n=1}^{\infty}\left(\frac{5}{6}\right)^n$ is converges or diverges. (02)

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

**a)** Prove that the set of rational numbers is not order complete. (05)

**b)** State and prove Bolzano Weierstrass theorem for sequences. (05)

**c)** Define: Supremum and Infimum and also find it for the following set: (04)

i) $\left\{\frac{1}{n}; n \in N\right\}$ ii) $\left\{\frac{(-1)^n}{n}; n \in N\right\}$

**Q-3 Attempt all questions (14)**

**a)** Using Sandwich theorem, prove that $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}}\right] = 1.$ (05)

**b)** Find $\underline{\lim}\, a_n$ and $\overline{\lim}\, a_n$ for the sequences $\{a_n\}$, where $a_n = (-1)^n\left(1+\frac{1}{n}\right)$. (05)

**c)** Show that every open interval contains a rational number. (04)

**Q-4 Attempt all questions** **(14)**

**a)** State and prove Cauchy's general principle of convergence for sequence. (07)

**b)** If $\{a_n\}$ is any sequence then prove the followings: (04)

(i) $\underline{\lim}(-a_n) = -\overline{\lim}\, a_n$ and (ii) $\overline{\lim}(-a_n) = -\underline{\lim}(a_n)$

**c)** Prove that $\lim_{n\to\infty} \frac{1+3+5+....+(2n-1)}{n^2} = 1$ (03)

**Q-5 Attempt all questions** **(14)**

**a)** State Leibnitz test for alternating series and using it test the convergence of the series $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + ....$ (05)

**b)** Show that the series $\frac{1\cdot 2}{3^2\cdot 4^2} + \frac{3\cdot 4}{5^2\cdot 6^2} + \frac{5\cdot 6}{7^2\cdot 8^2} + ...$ is convergent. (05)

**c)** Show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$ by using the definition. (04)

**Q-6 Attempt all questions** **(14)**

**a)** State and prove D'Almbert's ratio test. (09)

**b)** Prove that the series $\frac{1}{2} - \frac{1}{2}\cdot\frac{1}{2^2} + \frac{1}{3}\cdot\frac{1}{2^3} - \frac{1}{4}\cdot\frac{1}{2^4} + ...$ is absolutely convergent. (05)

**Q-7 Attempt all questions** **(14)**

**a)** Show that the geometric series $1 + r + r^2 + ...$ is (07)

i) Convergent if $|r| < 1$, ii) divergent if $r \geq 1$, iii) finitely oscillating if $r = -1$ and iv) Infinitely oscillating if $r < -1$.

**b)** Test the convergence of the series $x + \frac{x^2}{2} + \frac{x^3}{3} + ....; x > 0$. (04)

**c)** Test the behavior of the series $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + .....$ (03)

**Q-8 Attempt all questions** **(14)**

**a)** Test the convergence of the series $1 + x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + ...., x > 0$ (07)

**b)** Show that $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$ is conditionally convergent. (05)

**c)** State Cesaro's theorem. (02)